Sec. 9.1 Identities, Expressions and Equations

Equations vs Identities:

An equation is something that can be solved for some values of the variable.

An identity is an equation that is true for ALL values of the variable.

Ex: The Pythagorean identity, $\cos^2\theta + \sin^2\theta = 1$, can be rewritten in terms of other trigonometric functions. Provided $\cos\theta \neq 0$, dividing through by $\cos^2\theta$ and $\sin^2\theta$ would give:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \qquad \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Horizontal shift and reflection identities:

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$ The graph of $\cos\theta$ shifted right $\frac{\pi}{2}$ is the same as the graph of $\sin\theta$.

$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$ The graph of $\sin\theta$ shifted left $\frac{\pi}{2}$ is the same as the graph of $\cos\theta$.

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ The graph of $\cos\theta$ shifted left $\frac{\pi}{2}$ and reflected horizontally is the same as $\sin\theta$.

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ The graph of $\sin\theta$ shifted left $\frac{\pi}{2}$ and reflected horizontally is the same as $\cos\theta$.

Ex: Use the identities $\sin(-t) = -\sin t$ and $\cos(\pi/2 - t) = \sin t$ to rewrite the following as a sinusoidal function and gives its amplitude, midline and period.

$$y = 2\sin t - 3\sin(-t) + 4\cos(\pi/2 - t)$$

$$= 2\sin t + 3\sin t + 4\sin t$$

$$y = 9\sin t$$

Amplitude = 9

Period = 2π

Midline: $y = 0$

Ex: Simplify the expression: $(2\cos t + 3\sin t)(3\cos t + 2\sin t) - 13\sin t\cos t$.

$6\cos^2 t + 4\sin t\cos t + 9\sin t\cos t + 6\sin^2 t - 13\sin t\cos t$

$6\cos^2 t + 6\sin^2 t$

$6(\cos^2 t + \sin^2 t)$

$6(1)$

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Ex: Simplify the expression: $\dfrac{\cos\theta - 1}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta + 1}$

$\left(\dfrac{\cos\theta+1}{\cos\theta+1}\right)\cdot\dfrac{\cos\theta-1}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta+1}\cdot\left(\dfrac{\sin\theta}{\sin\theta}\right)$

$\dfrac{\cos^2\theta - 1 + \sin^2\theta}{\sin\theta(\cos\theta+1)}$

$\dfrac{\cos^2\theta + \sin^2\theta - 1}{\sin\theta(\cos\theta+1)}$

$\dfrac{1-1}{\sin\theta(\cos\theta+1)}$

$\dfrac{0}{\sin\theta(\cos\theta+1)} = 0$

Ex: Suppose that $\cos\theta = 2/3$ and $3\pi/2 \le \theta \le 2\pi$. Find $\sin\theta$ and $\tan\theta$. Q4

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{2}{3}\right)^2 + \sin^2\theta = 1$

$\frac{4}{9} + \sin^2\theta = 1$

$\sin^2\theta = \frac{5}{9}$

$\sin\theta = \pm\frac{\sqrt{5}}{3}$

$\sin\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= -\frac{\sqrt{5}}{3} \div \frac{2}{3}$

$= -\frac{\sqrt{5}}{3}\cdot\frac{3}{2}$

$\tan\theta = -\frac{\sqrt{5}}{2}$

Ex: Solve : $2\sin^2 t = 3 - 3\cos t$ for $0 \le t \le \pi$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$2(1-\cos^2 t) = 3 - 3\cos t$

$2 - 2\cos^2 t = 3 - 3\cos t$

$0 = 2\cos^2 t - 3\cos t + 1$

$0 = (2\cos t - 1)(\cos t - 1)$

$2\cos t - 1 = 0$ $\quad$ $\cos t - 1 = 0$

$2\cos t = 1$ $\quad$ $\cos t = 1$

$\cos t = \frac{1}{2}$ $\quad$ $t = \cos^{-1}(1)$

$t = \cos^{-1}\left(\frac{1}{2}\right)$ $\quad$ $t = 0$

$t = \frac{\pi}{3}$

Double Angle Formula for Sine: $\boxed{\sin 2\theta = 2 \sin \theta \cos \theta}$

Ex: Find all solutions to the equation $\sin 2t = 2 \sin t$ on the interval $0 \le t \le 2\pi$.

$2\sin t\cos t = 2\sin t$

$2\sin t\cos t - 2\sin t = 0$

$2\sin t(\cos t - 1) = 0$

$2\sin t = 0 \qquad \cos t - 1 = 0$

$\sin t = 0 \qquad \cos t = 1$

$\boxed{t = 0, \pi, 2\pi} \qquad t = 0, 2\pi$

Double Angle Formulas for Cosine and Tangent:

The double-angle formula for the cosine can be written in three forms:

$\cos 2\theta = 1 - 2\sin^2\theta \qquad \cos 2\theta = 2\cos^2\theta - 1 \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$

$\boxed{\tan 2\theta = (2\tan\theta)/(1 - \tan^2\theta)}$

Ex: Solve $\sin 2t = \sqrt{2}\sin(t + \pi/2)$ for one period.

$2\sin t\cos t = \sqrt{2}\cos t$

$2\sin t\cos t - \sqrt{2}\cos t = 0$

$\cos t(2\sin t - \sqrt{2}) = 0$

$\cos t = 0 \qquad 2\sin t - \sqrt{2} = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 2\sin t = \sqrt{2}$

$\sin t = \frac{\sqrt{2}}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

$\boxed{t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}}$

Ex: Use a graphing calculator to check to see if the following equation is an identity. If it is, prove it algebraically. $\tan\theta + \frac{1}{\tan\theta} = \frac{1}{\sin\theta\cos\theta}$

$\frac{\sin\theta}{\cos\theta} + \cot\theta$

$\left(\frac{\sin\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta}$

HW: p 375-377 #1,2,5-8,10,12,18, 23,26-~~29~~ 28,32,39,41